

# Simulated Moving-Bed Reactor: Reactive–Separation Regions

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*This work focuses on the design of a simulated moving-bed reactor (SMBR) where reaction  $A \rightarrow B + C$  takes place. The reactive–separation regions were determined for two reactive systems: (1) inversion of sucrose, with enzyme introduced in the unit through the eluent stream and Michaelis–Menten reaction kinetics, and (2)  $A \rightarrow B + C$  reaction, with immobilized enzyme and linear reaction kinetic law. In both systems the reaction species exhibit linear adsorption isotherms. The steady-state equivalent true moving-bed reactor (TMBR) analogy was applied in the algorithm used for determination of the reactive–separation regions. The influence of the mass-transfer limitation, reaction rate, product purities, reactant Henry constant, and SMBR configuration on the shape and position of reactive–separation regions was analyzed. It was shown that in certain conditions the reactive–separation regions extended out of the separation regions obtained for nonreactive SMB for product (B and C) separation. © 2005 American Institute of Chemical Engineers AIChE J, 51: 2737–2751, 2005*

**Keywords:** simulated moving-bed reactor, reactive–separation region, irreversible reaction, sucrose inversion, Michaelis–Menten

## Introduction

Reactive separations combine the generally separated and sequential unit operations of reaction and separation into a simultaneous single operation.<sup>1</sup> The main advantages of the reactive–separation processes are reduction of the operating and capital cost, as well as improvement of the reaction and separation efficiency. The simulated moving-bed reactor (SMBR) technology is a combination of the chemical or biochemical reaction with a simulated moving bed (SMB) chromatographic separator. The SMBR is a continuous countercurrent process that brings advantages in view of the separation/reaction, as increased conversion and/or product purity, increased driving force by countercurrent operation, and overcomes the difficulties connected to the solid abrasion and handling by simulation of the countercurrent flow. The poten-

tial applications of this technology are in (1) equilibrium-limited reversible reaction where the removal of products as they are formed allows achieving conversions well beyond equilibrium values; (2) reactions in series or in parallel, where the desired intermediate species could be separated selectively; and (3) reactions with inhibiting or poisoning product, where its removal from the reaction medium could promote yield improvement.

The first application of reactive SMB in a zeolite-catalyzed alkylation reaction dates back to the 1970s.<sup>2</sup> In the last decade the SMBR has been the subject of considerable attention mainly because of the increased interest in the area of process intensification and successful entrance of the SMB technology in the pharmaceutical industry.

The laboratory-scale SMBR has been used effectively to carry equilibrium-limited reversible reaction in the liquid phase, such as esterifications,<sup>3–7</sup> etherifications,<sup>8</sup> alkylations,<sup>9</sup> and acetalization<sup>10</sup>; as well as gas-phase reactions: hydrogenations,<sup>11,12</sup> partial oxidation of methane,<sup>13</sup> and oxidative coupling of methane.<sup>11,14,15</sup> The biochemical reactions involving

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sugars have been also investigated extensively, as glucose isomerization,<sup>16-19</sup> inversion of sucrose by enzyme invertase,<sup>20-22</sup> production of high molecular weight dextran,<sup>21</sup> enzymatic conversion of glucose to glucose-6-phosphate,<sup>23</sup> hydrolysis of lactose and maltose,<sup>24,25</sup> and the production of lactosucrose.<sup>26</sup>

Even though a reasonable amount of experimental and numerical studies on SMBR have been published, there is still no reported application of SMBR in the chemical process industry. The determination of SMBR optimal design parameters and operating conditions is decisive in the study of the process viability at the industrial level.

Few studies have been reported on the design and optimization of SMBR, based on either simulation<sup>22,27</sup> or single<sup>22,28</sup> or multiobjective optimization.<sup>18,29-31</sup>

Dünnebier et al.<sup>28</sup> presented a novel optimization and design strategy for an SMBR based on a rigorous process model and a detailed cost function. Azevedo and Rodrigues<sup>22</sup> adopted the algorithm proposed by Biressi et al.<sup>32</sup> to design both the operating and the geometric parameters of a reactive SMB for sucrose inversion in the presence of mass-transfer resistance.

Design criteria for both irreversible ( $A \rightarrow B + C$ ) and reversible reaction ( $A \rightleftharpoons B + C$ ), where the reactant adsorption affinity is between those of the products, based on the TMBR equivalence and the equilibrium theory have been developed.<sup>33,34</sup> A novel analytical solution of equivalent TMBR, for the reaction of type  $A \rightarrow B + C$ , in the presence of intraparticle mass-transfer resistance described by linear driving force (LDF) model has been recently applied in the design of the SMBR reactive-separation regions.<sup>35</sup>

The design of an SMBR system is associated with several important objectives [productivity, eluent (desorbent) consumption, adsorbent/catalyst inventory] that require optimization, along with the constraints (product purity, conversion, recovery) that must be satisfied at the same time. It has been shown that genetic algorithms can be useful in the optimization of these complex units.<sup>18,29-31</sup> In multiobjective optimization there is an entire set of equally good optimal solutions (Pareto optimal solutions). Often, it remains difficult to draw general conclusions about the optimal design of the SMBR units because the choice of one solution over the other requires additional knowledge of the problem.

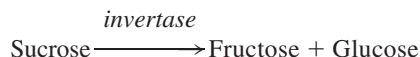
In this work the design of SMBR where the reaction  $A \rightarrow B + C$  takes place was studied. The steady-state equivalent true moving-bed reactor (TMBR) analogy was implemented in the algorithm used for determination of the reactive-separation regions. Two reactive systems were considered: (1) inversion of sucrose, with enzyme introduced in the unit through the eluent stream and Michaelis-Menten reaction kinetics, and (2)  $A \rightarrow B + C$  reaction, with immobilized enzyme and linear reaction kinetic law. In both systems the reaction species exhibit linear adsorption isotherms.

The design of the SMBR for sucrose inversion is presented in the first part of this paper. The bilinear driving force (bi-LDF) equivalent TMBR model is used to study the influence of mass-transfer resistance, product purities, SMBR configuration, and enzyme concentration on the reactive-separation regions of SMBR for sucrose inversion. The effect of the enzyme concentration and reactant Henry constant on the reactive-separation regions was also studied in the absence of the mass-transfer resistance.

In the second part, the design of SMBR for a general type of irreversible reaction  $A \rightarrow B + C$ , in the absence of mass transfer, is discussed. Special attention is given to the influence of the reactant (A) adsorption affinity on the reactive-separation regions.

## SMBR for Sucrose Inversion

The SMBR for the enzymatic inversion of sucrose and separation of fructose from glucose was considered. The inversion of sucrose by enzyme invertase is an irreversible reaction:



The reaction rate is not influenced by product accumulation. Meurer et al.<sup>36</sup> showed that the SMBR, when applied even for irreversible reactions, provides better conversion and product purities than the conventional processes. Also, the simultaneous inversion and product separation overcomes the problems associated with substrate inhibition.<sup>21</sup>

The SMBR, presented in Figure 1a, consists of a set of columns packed with cation-exchange resin (DOWEX Mono-sphere 99/Ca, by Sigma) and arranged in a closed circuit, with two inlet streams (feed and eluent) and two outlet streams (extract and raffinate). Sucrose (A) solution in water is used as feed and water is used as eluent. The enzyme is introduced in the system diluted in the eluent (buffer solution). The fructose (B) is collected in the extract as a more strongly adsorbed component and glucose (C) is collected in the raffinate as less strongly adsorbed component. At regular time intervals, the introduction and withdrawal points are advanced one bed ahead in the direction of fluid flow; the countercurrent motion of the solid adsorbent is simulated and the solid velocity is equal to the length of a column divided by the switching time. When the number of switches is equal to the number of columns one cycle is completed.

The SMBR principle of operation may be more clearly understood by considering the equivalent representation of a true moving-bed reactor (TMBR). In the TMBR (Figure 1b) the inlet and outlet points are kept unchanged and the solid phase (adsorbent) really moves in the direction opposite to that of the fluid phase. The TMBR representation also provides a simpler model for the purposes of process prediction and design.

## Mathematical model

The bi-LDF steady-state equivalent TMBR model has been used to describe the SMBR unit behavior. The model assumes axially dispersed flow for the fluid phase and plug flow for the solid phase, diffusion through the external film, and intraparticle mass transfer represented by means of a bilinear driving force.<sup>22</sup> The reaction of sucrose inversion is described by the Michaelis-Menten equation. Sucrose is assumed to react in the interparticle fluid phase and at the surface of the resin as a result of the immobilized enzyme adsorbed at its exterior surface. The reaction products, glucose and fructose, have linear adsorption isotherms. Sucrose adsorption into the adsor-

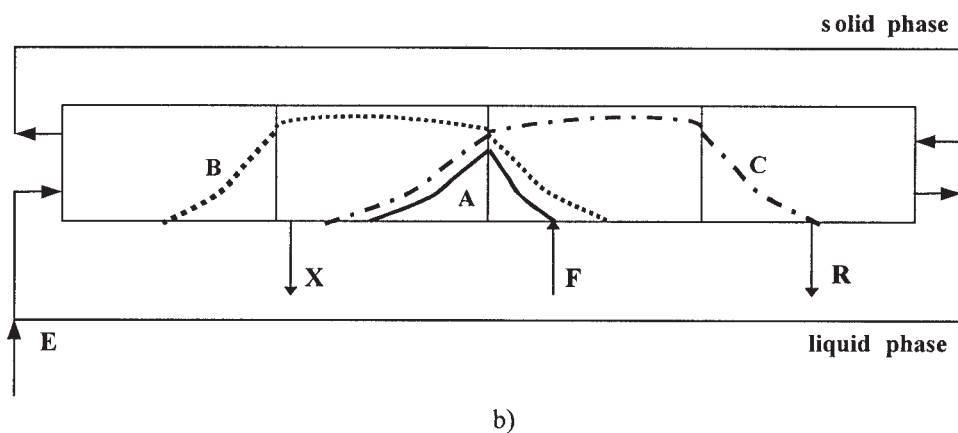
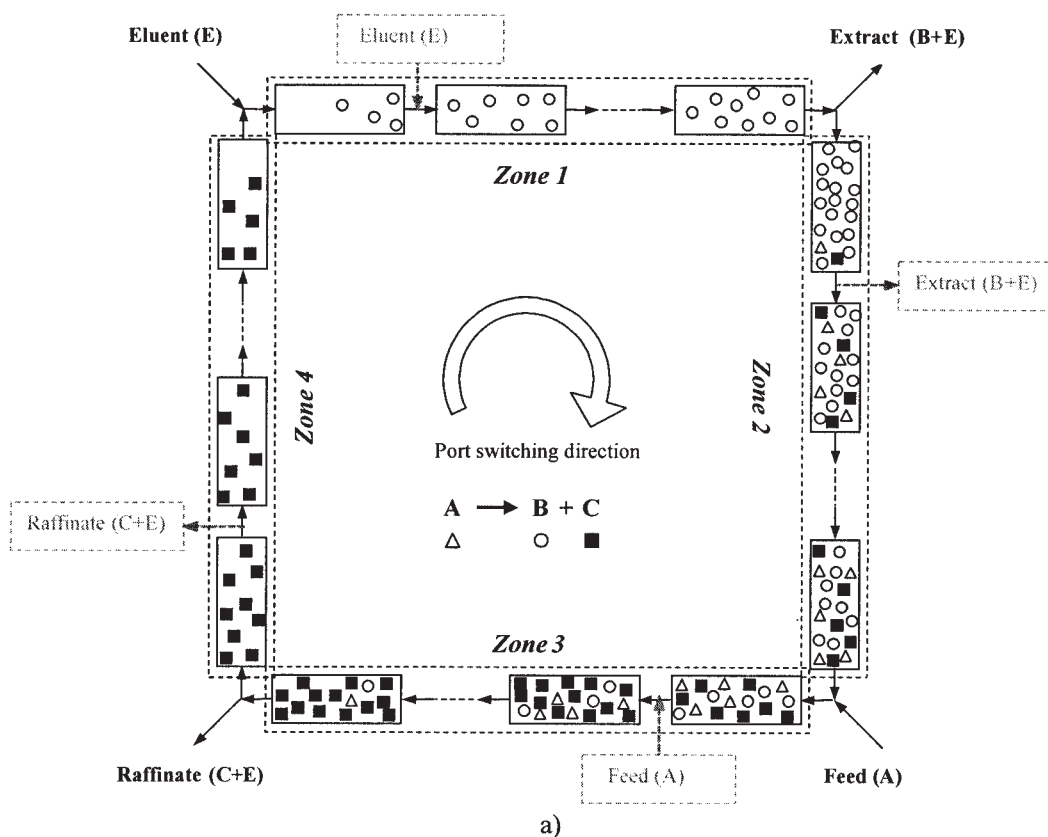


Figure 1. (a) SMBR schematic diagram and (b) TMBR schematic diagram.

bent particles was not considered in the model because it has been observed experimentally that the adsorption constant of sucrose is nearly equal to the particle porosity.<sup>37</sup>

The model equations are summarized in Table 1. All mass-balance equations are written in terms of mass rather than moles. Model equations were numerically solved by gPROMS.<sup>38</sup> The system of ordinary differential and algebraic equations (DAE) was solved by DASOLV solver incorporated in gPROMS. The axial domain is discretized using a third-order orthogonal collocation method in finite elements (OCFEM). In the case of significant mass-transfer limitation 20 finite elements per column were used; other-

wise, 200 elements per column were considered. For a typical simulation, CPU times of about 0.5 s in the case of mass-transfer limitation and 20 s in an absence of mass-transfer limitations were required on Pentium IV 2300 MHz processor with 2 GB RAM memory.

#### *Reactive-separation region in the presence of mass-transfer limitation*

The reactive-separation regions of the SMBR for sucrose inversion were determined with the bi-LDF steady-state equivalent TMBR model. The influence of the (1) product purities

**Table 1. Bi-LDF Steady-State Equivalent TMBR Model Equations**

Mass balance in the interparticle liquid phase in zone $j$	
$\frac{\gamma_j}{Pe_j} \frac{d^2 C_{i,j}}{dx^2} - \gamma_j \frac{dC_{i,j}}{dx} - \nu \frac{Bi_{mj}}{Bi_{mj} + 5} \alpha_{pi,j} (C_{i,j} - \langle C_p \rangle_{i,j}) + \alpha_{rj} (1 + \nu K_{Enz}) \left( \frac{\sigma_i R_j}{k_r} \right) = 0 \quad (i = A, B, C)$	
$\frac{1}{Pe_j} \frac{d^2 C_{i,j}}{dx^2} - \frac{dC_{i,j}}{dx} = 0 \quad (i = Enz)$	
where	
$R_j = k_r \frac{C_{s,j} \times C_{e,j}}{K_{mm} + C_{s,j}} \quad \sigma_i = \begin{cases} -1 & \text{if } i \text{ is } A \\ +0.526 & \text{if } i \text{ is } B \text{ or } C \end{cases}$	
Mass balance in the liquid phase within particle pores in zone $j$	
$\frac{\partial \langle C_p \rangle_{i,j}}{\partial x} + \frac{Bi_{mj}}{5 + Bi_{mj}} \frac{\alpha_{pi,j}}{\varepsilon_p} (C_{i,j} - \langle C_p \rangle_{i,j}) - \frac{\alpha_{\mu i,j}}{\varepsilon_p} [K_i \langle C_p \rangle_{i,j} - \langle \langle q \rangle \rangle_{i,j}] = 0 \quad (i = A, B, C)$	
Mass balance in the solid phase within the adsorbent particles in zone $j$	
$\frac{\partial \langle \langle q \rangle \rangle_{i,j}}{\partial x} + \alpha_{\mu i,j} [K_i \langle C_p \rangle_{i,j} - \langle \langle q \rangle \rangle_{i,j}] = 0 \quad (i = A, B, C)$	
Boundary conditions for a given zone $j$ are	
$C_{i,j}^n = C_{i,j}(0) - \frac{1}{Pe_j} \frac{\partial C_{i,j}}{\partial x} (1) = 0 \quad (i = A, B, C, Enz)$	
$\langle C_p \rangle_{i,j}(1) = \langle C_p \rangle_{i,j+1}(0) \quad \langle \langle q \rangle \rangle_{i,j}(1) = \langle \langle q \rangle \rangle_{i,j+1}(0) \quad (i = A, B)$	
Dimensionless groups	
$\gamma_j = \frac{U'_{Fj}}{U_S} \quad Pe_j = \frac{U'_{Fj} L_j}{D_{axj}} \quad \nu = \frac{(1 - \varepsilon)}{\varepsilon} \quad \sigma_i = \frac{\text{Mol. weight of } B \text{ (or } C)}{\text{Mol. weight of } A}$	
$\alpha_{pi,j} = \frac{k_{pi} L_j}{U_S} \quad \alpha_{rj} = \frac{k_r L_j}{U_S} \quad \alpha_{\mu i,j} = \frac{k_{\mu i} L_j}{U_S} \quad Bi_{mj} = \frac{k_{fj} R_p}{D_{pe}}$	
Node mass balances	
<i>Eluent node</i>	<i>Feed node</i>
$C_{i,1}^n = \frac{Q'_4}{Q'_1} C_{i,4}(1) + \frac{Q_E}{Q'_1} C_{i,E}$	$C_{i,3}^n = \frac{Q'_2}{Q'_3} C_{i,2}(1) + \frac{Q_F}{Q'_3} C_{i,F}$
<i>Extract node</i>	<i>Raffinate node</i>
$C_{i,2}^n = C_{i,1}(1)$	$C_{i,4}^n = C_{i,3}(1)$

and (2) enzyme concentration in the eluent, on the reactive-separation region size and position was studied. In both cases the required conversion was set at the minimum of 99%. The expressions used to calculate the product purities and reactant conversion are given in Table 2.

Adsorption equilibrium data, mass transfer, and reaction parameters for sucrose inversion and glucose-fructose adsorp-

tive separation, provided by Azevedo and Rodrigues,<sup>22</sup> are presented in Table 3.

The values of the  $\gamma_1$  and  $\gamma_4$  were calculated according to the equilibrium theory for linear equilibrium nonreactive systems<sup>39</sup>:

$$\gamma_1 > \nu(K_B + \varepsilon_p) = 0.795$$

**Table 2. Expressions Used for SMBR Performance**

Performance	Expression
Extract purity (%)	$PUX = 100 \times \frac{C_{B,E}}{C_{A,E} + C_{B,E} + C_{C,E}}$
Raffinate purity (%)	$PUR = 100 \times \frac{C_{C,R}}{C_{A,R} + C_{B,R} + C_{C,R}}$
Conversion (%)	$X = 100 \times \frac{C_{A,F} Q_F - (C_{A,X} Q_X + C_{A,R} Q_R)}{C_{A,F} Q_F}$
Enzyme productivity (kg <sub>A</sub> /g <sub>Enz</sub> )	$PRE = \frac{C_{A,F} Q_F \times X/100}{C_{Enz,E} Q_E}$
Adsorbent productivity (kg m <sup>-3</sup> h <sup>-1</sup> )	$PRA = \frac{C_{A,F} Q_F \times X/100}{V_{ads}}$

**Table 3. Bi-LDF Steady-State Equivalent TMBR Model Input Parameters**

Model Parameters	Operating Conditions	Columns
$Pe = 500/\text{column}$ $Bi_m = 500$ $k_r = 50.32 \text{ min}^{-1}$ $K_{mm} = 23 \text{ g/L}$ $k_{pA} = 1.7 \text{ min}^{-1}$ $k_{pB(C)} = 2.5 \text{ min}^{-1}$ $k_{\mu A} = 0 \text{ min}^{-1}$ $k_{\mu B(C)} = 1.5 \text{ min}^{-1}$ $K_A = 0$ $K_B = 0.43$ $K_C = 0.17$ $K_{Enz} = 4$ $\varepsilon_p = 0.1$	$T = 55^\circ\text{C}$ $pH = 4.5$ $C_F = 80 \text{ g/L}$ $C_{Enz} = 25\text{--}250 \text{ mg/L}$ $t^* = 3.4 \text{ min}$ $Q_{Rec} = Q_4 = 24 \text{ mL/min}$ $\gamma_1 = 0.953$ $\gamma_4 = 0.325$	$D_b = 2.6 \text{ cm}$ $L_b = 29 \text{ cm}$ Configuration: 3–2–5–2 $d_p = 320 \text{ }\mu\text{m}$

$$\gamma_4 < \nu(K_C + \varepsilon_p) = 0.405$$

Safety margins of 1.2 and 1.25 were applied for  $\gamma_1$  and  $\gamma_4$ , respectively. The  $\gamma_1$  and  $\gamma_4$  were set on 0.953 and 0.325, respectively.

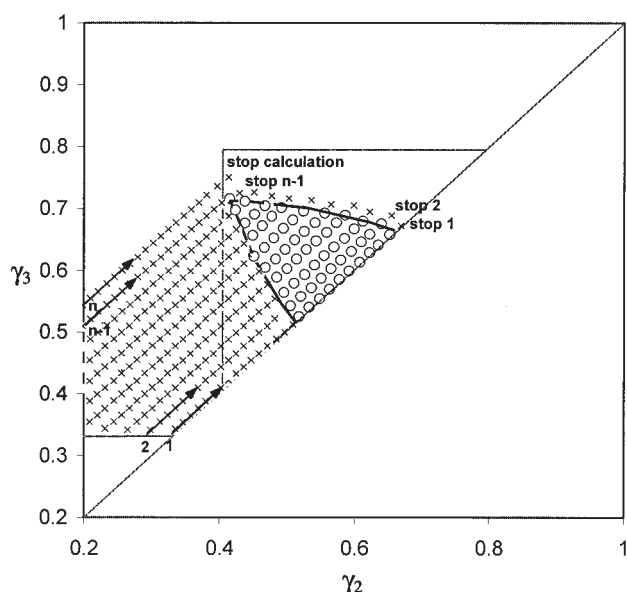
The algorithm used in the determination of the reactive-separation regions is presented in Figure 2. The steady-state TMBR model was successively solved for several values of  $\gamma_2$  and  $\gamma_3$  within the region between the diagonal  $\gamma_2 = \gamma_3$ , the horizontal line  $\gamma_3 = 0.325$ , and the  $\gamma_3$  axis. The diagonal  $\gamma_2 = \gamma_3$  corresponds to a zero feed flow rate, and thus  $\gamma_3$  must be higher than  $\gamma_2$ . The horizontal branch  $\gamma_3 = 0.325$  corresponds to a zero raffinate flow rate.

The starting point (first point in step 1) in the algorithm corresponds to  $Q_F = 0.01 \text{ mL/min}$  and  $\gamma_2 = 0.325$ . The feed flow rate was kept constant and the flow rate in zone 2 was increased gradually starting from the value  $\gamma_2 = 0.325$ . For any value of flow rate in zone 2 the flow rate in zone 3 was calculated using the mass balance for the feed node, and was translated to the  $\gamma_3$  form. For each pair of  $(\gamma_2, \gamma_3)$  the conversion and product purities were estimated. The values that satisfy the conversion and purity criteria were selected to build the reactive-separation region. The values that did not accomplish these requirements were discarded. After the first set of simulations was finished (step 1), the feed flow rate was in-

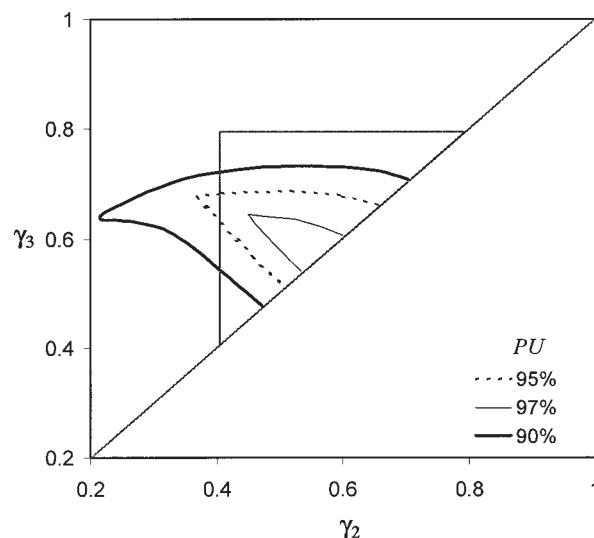
creased to the next higher value and the same procedure was repeated. The simulation procedure ends with the last (highest) value of the feed flow rate, which gives required product purities and conversion. Above that feed flow rate value the conversion and purity requirements cannot be fulfilled for any pair of values of  $(\gamma_2, \gamma_3)$ .

**Influence of the Product Purities on Reactive-Separation Regions.** The influence of the required product purities on the SMBR reactive-separation regions was studied. The SMBR unit configuration, model parameters, and operating conditions are presented in Table 3. The enzyme concentration used in this study was 50 mg/L. The product purities requirements were set on 90, 95, and 97% with a reactant minimum conversion of 99%. The obtained reactive-separation regions are presented in Figure 3. The operation of the SMBR at the upper border of each reactive-separation region would lead to raffinate with the required purity (90, 95, and 97%) and extract with purity higher than required. On the other hand, the operation in the lower border leads to extract with the required purity and raffinate with purity higher than required. The conversion was complete (100%) on both borders for each of the reactive-separation regions presented in Figure 3.

The reactive-separation regions expand with decrease of the purity requirements and, in the case of purities of 90 and 95%, the region extends its borders out of the separation region



**Figure 2. Algorithm for determination of the reactive-separation regions.**



**Figure 3. Reactive-separation regions for different product purities in the presence of mass-transfer resistance.**

$$X_{\min} = 99\% \text{ and } C_{Enz} = 50 \text{ mg/L.}$$

defined for the case of nonreactive separation and no mass-transfer limitation. It is also interesting to observe that the vertex (equal purity in extract and raffinate) moves to lower rather than higher values of  $\gamma_3$  in the case of 90% product purity restriction.

To better understand this behavior it is important to point out two peculiarities of the SMBR system considered here:

(1) The reaction occurs mainly in zone 3. The sucrose passing to zone 2 is strictly a result of the presence of mass-transfer limitations.

(2) The enzyme is introduced in the system with the eluent. Therefore, there are two zones of different concentrations of enzyme in the system: (a) higher enzyme concentration in zones 1 and 2; and (b) lower enzyme concentrations in zones 3 and 4, resulting from the dilution of the enzyme by the introduction of the feed stream.

In the case of low feed flow rate the concentration of the enzyme is sufficient for complete and almost instantaneous sucrose conversion just at the beginning of zone 3 (only a small part of this zone is used for reaction), and SMBR behaves closely to a nonreactive SMB for separation of fructose/glucose feed mixture.

When the feed flow rate increases, the reactive-separation region extends out of the nonreactive SMB separation region. A higher feed flow rate means a higher quantity of sucrose to be converted and lower enzyme concentration in zone 3. Consequently, the flow rate in zone 3 ( $\gamma_3$ ) needs to be lower to achieve both high conversion and adsorption of fructose. Also, the values of  $\gamma_2$  violate the restrictions defined for a nonreactive SMB, as a result of the allowed product contamination. The glucose would not be completely desorbed in zone 2 and would contaminate the extract. When the allowed contamination increases, from 3 to 10%, the reactive-separation regions expand to the lower values of  $\gamma_2$ .

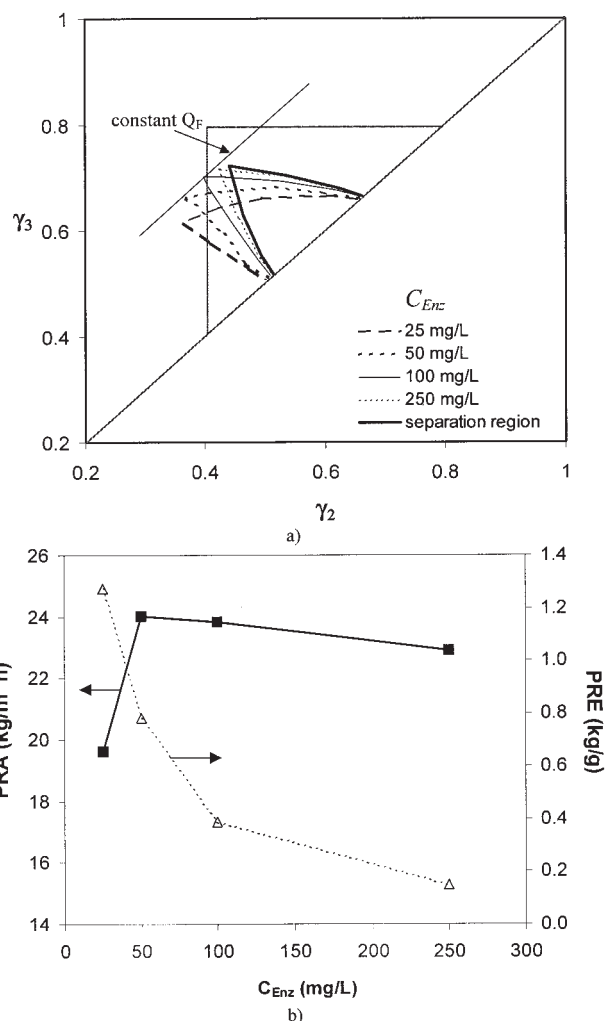
**Influence of the Enzyme Concentration on Reactive-Separation Regions.** The influence of the enzyme concentration was studied for 95% product purities restriction and minimum conversion of 99%. For this study the enzyme concentration was varied between 25 and 250 mg/L.

The reactive-separation regions for different enzyme concentrations are presented in Figure 4a. In the same figure, the region obtained in the case of nonreactive SMB for separation of a fructose/glucose mixture, assuming that the unit is fed with completely inverted sucrose (42.08 g/L of glucose and fructose), is also presented.

The vertices move to the region of lower values of  $\gamma_2$  and  $\gamma_3$  with decrease of the enzyme concentration in the eluent. The reason for this behavior is connected to the feed flow rate and enzyme concentration and was explained in detail in the previous section.

The reactive-separation region approaches the nonreactive separation region with increasing enzyme concentration. In the case of sufficiently high invertase concentration the reaction is very fast and sucrose conversion is complete at the entrance in zone 3; then the SMBR behavior approaches that of the SMB for separation of glucose/fructose mixture.

The maximum feed can be processed with enzyme concentrations between 50 and 250 mg/L (see Figure 4a). In all these cases the feed flow rate in the vertex is almost the same. Besides the maximum throughput, product purities, and conversion, the enzyme and adsorbent performances are of crucial

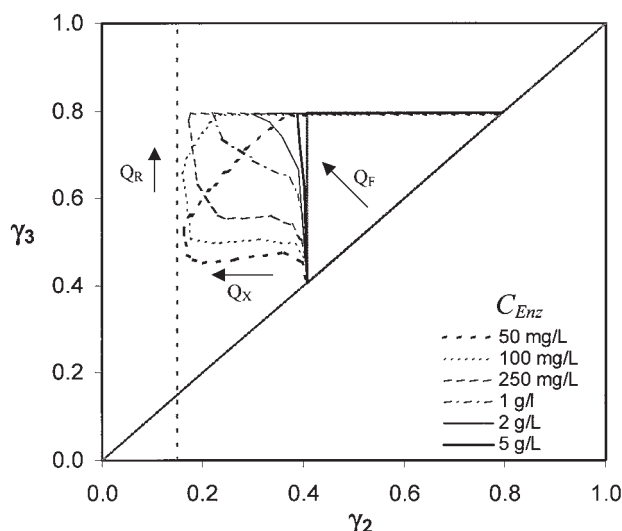


**Figure 4. (a) Reactive-separation regions for different enzyme concentrations in the presence of mass-transfer resistance; (b) enzyme productivity ( $PRE$ ) and adsorbent productivity ( $PRA$ ) in the vertexes of the reactive-separation regions for different enzyme concentrations ( $PUR = 95\%$ ,  $PUR = 95\%$ , and  $X_{min} = 99\%$ ).**

interest. The enzyme and adsorbent productivities ( $PRE$  and  $PRA$ , respectively) in the vertexes for different enzyme concentrations are presented in Figure 4b. It can be observed that the best performances would be expected for an enzyme concentration around 40 mg/L.

#### **Reactive-separation region in the absence of mass-transfer limitation**

To better understand the factors that govern the size and position of reactive-separation regions the SMBR for sucrose inversion was studied in the absence of mass-transfer limitation. The bi-LDF model was used and mass transfer and axial dispersion were eliminated by multiplying the mass-transfer parameters and the Peclet ( $Pe$ ) number by a factor of 100. The product purity requirements were restricted to 99% because it was previously concluded that one of the reasons for the



**Figure 5. Reactive-separation regions for different enzyme concentrations in absence of mass transfer resistance ( $P_{UX} = 99\%$ ,  $P_{UR} = 99\%$ , and  $X_{\min} = 99\%$ ).**

extension of the borders of SMBR reactive-separation region out of the nonreactive SMB region could be the relaxation of product purities requirements. The product purities of 99% are possible only for very high conversions ( $X > 99\%$ ). The product conversion requirement was kept at the minimum of 99%. Besides, in most cases the conversion in the reactive-separation region border was between 99.7 and 100%. The same net flow rates in zones 1 and 4 as in the case with mass-transfer limitation were used ( $\gamma_1 = 0.953$  and  $\gamma_4 = 0.325$ ).

**Influence of the Enzyme Concentration on Reactive-Separation Regions.** The reactive-separation regions for sucrose inversion in the absence of mass-transfer resistance were determined for enzyme concentrations between 50 mg/L and 5 g/L. The obtained regions are presented in Figure 5. For all enzyme concentrations the reactive-separation regions expand out of the nonreactive separation region (gray triangle), calculated from the equilibrium theory for an SMB for reaction products (glucose and fructose) separation. The minimum  $\gamma_2$  value and maximum  $\gamma_3$  value reached by the reactive-separation regions correspond to values calculated with the equilibrium theory,  $\gamma_{2\min} = \nu(K_A + \varepsilon_p) = 0.150$  and  $\gamma_{3\max} = \nu(K_B + \varepsilon_p) = 0.795$ , respectively. All reactive-separation regions share the same upper border, adjacent to the region of impure raffinate. In the case of enzyme concentrations between 50 and 100 mg/L the upper border of the reactive-separation regions is cut along a line that corresponds to almost constant feed flow rate ( $\gamma_3 - \gamma_2 \approx \text{const}$ ). With increasing enzyme concentration the regions expand to  $\gamma_3 = 0.795$ . The lower border of the reactive-separation regions, adjacent to the region of impure extract, moves to higher values of  $\gamma_2$  and  $\gamma_3$  with increasing enzyme concentration. For  $C_{\text{Enz}} > 5$  g/L the enzyme concentration in zone 3, near the feed port, is high enough to instantaneously convert all sucrose introduced in the unit and therefore the SMBR unit would behave as an SMB unit fed with glucose/fructose mixture.

In the following the reasons that lead to the shape and

position of the obtained reactive-separation regions will be discussed.

There are two possible reasons for the extension of the reactive-separation region to lower values of  $\gamma_2$  relative to those calculated for the equivalent nonreactive SMB. The first reason is the rate of the reaction; that is, in the SMBR for sucrose inversion without mass-transfer limitation the SMBR design is governed primarily by the reaction rate. When the reaction is slow both glucose and fructose are produced at some distance from the feed port. This situation can be associated with an SMB in which the point of introduction of the glucose/fructose mixture is moved along zone 3 and part of zone 3 behaves as an extension of zone 2, that is, plays the role of zone 2 (adsorption of fructose). The second reason can be the negligible contamination of 1% of the products allowed. The conversion and product purities were calculated for the operating points laying on a line with constant  $\gamma_3 (=0.622)$ , which crosses the reactive-separation region obtained for  $C_{\text{Enz}} = 250$  mg/L. The conversion and product purities for each point on the line and  $C_{\text{Enz}} = 250$  mg/L are presented in Figure 6. The raffinate and extract purities of 100% can be obtained just for the values of  $\gamma_2 > 0.405$  (that is,  $\gamma_{2\min}$  determined according to the equilibrium theory for nonreactive SMB for glucose/fructose separation). The raffinate purity decreases from 100 to 99% when the  $\gamma_2$  value decreases from  $\gamma_2 = 0.405$  to the  $\gamma_2$  value on the border of the reactive-separation region.

The reactive-separation region for  $C_{\text{Enz}} = 50$  mg/L and  $C_{\text{Enz}} = 250$  mg/L together with some operating points inside and outside the regions are presented in Figure 7. The steady-state concentration profiles in the selected points were analyzed to better understand the grounds for the region's shape.

The concentration profiles of the reaction species (A, B, and C) and the enzyme in the operating points 1, 2, and 3 calculated for  $C_{\text{Enz}} = 50$  mg/L are presented in Figure 8a. For all these points the flow rate in zone 2 is the same (equal  $\gamma_2$ ). Moving from point 1 to point 3, the feed flow rate increases, leading to an increase of the concentration of sucrose at the beginning of zone 3 and the residence time decreases because the flow rate in zone 3 increases with increasing feed flow rate. Also, although the enzyme concentration in the eluent is the same, the enzyme concentration in the SMBR decreases going from point 1 to point 3. This decrease in the enzyme concentration is more visible in the reaction zone (zone 3). The rate of the reaction at the entrance of zone 3 decreases in direction to point 3. In the conditions of point 1, because the reaction is fast and the residence time sufficiently long, the complete conversion of sucrose occurs near the feed port. However, the flow rate in zone 3 is not high enough to prevent the passage of glucose in zone 2 and contamination of the extract. This contamination is reduced and products with required purities (99%) are obtained in point 2. At this point the reaction is slower as a result of lower enzyme concentration in zone 3, although all sucrose is still converted until the middle of zone 3. The reaction occurs at a longer distance from the feed port and prevents the passage of glucose in zone 2. In point 3 the reaction becomes so slow that all zone 3 is used for the sucrose inversion. The sucrose is not completely converted until the raffinate port; both products are being produced and contamination of the raffinate by fructose and sucrose can be observed. Only point 2 is inside the reactive-separation region for  $C_{\text{Enz}} = 50$  mg/L.

The internal concentration profiles for the operating condi-

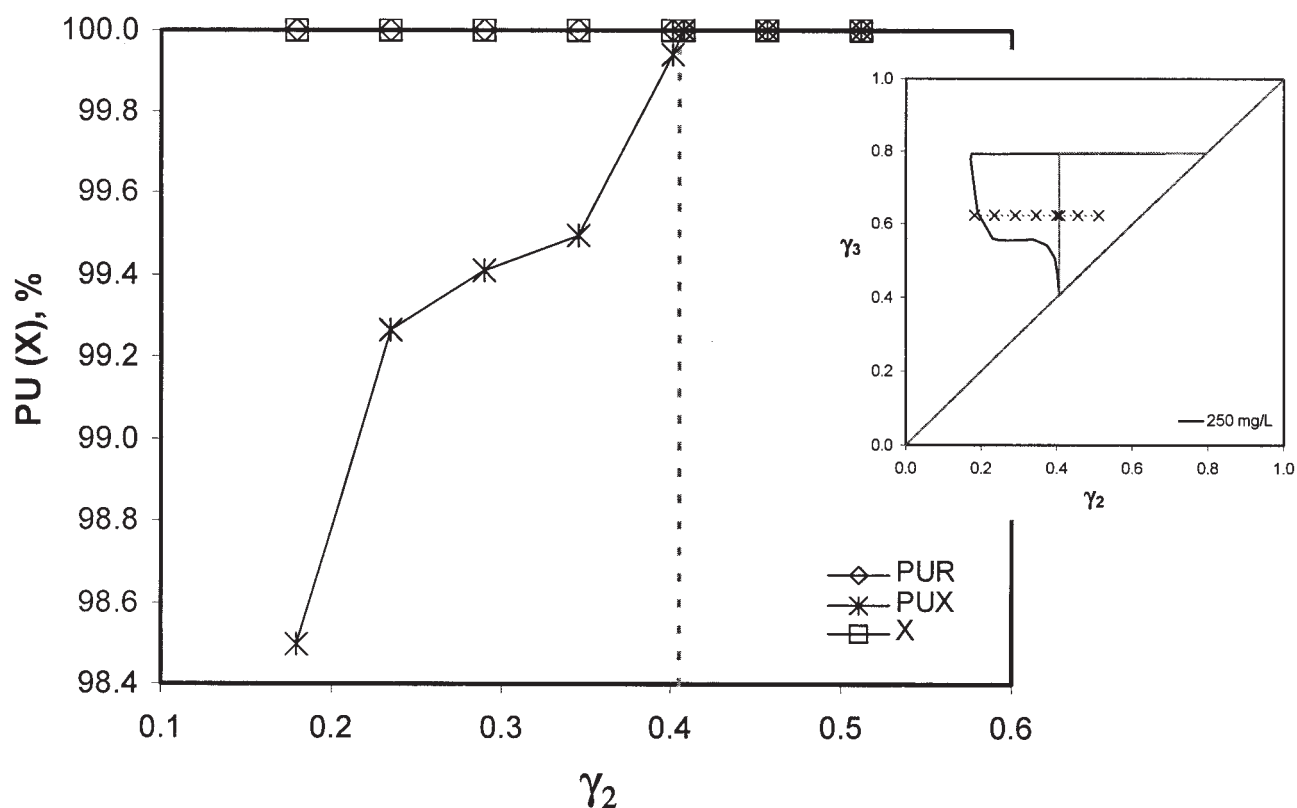


Figure 6. Conversion and product purities in selected operating points along the dashed line corresponding to  $\gamma_3 = 0.622$ , for  $C_{ENZ} = 250$  mg/L.

tions in points 2 and 3 were also calculated for  $C_{ENZ} = 250$  mg/L (see Figure 8b). The sucrose concentration at the feed port and the residence time are the same as in the previous case ( $C_{ENZ} = 50$  mg/L). In point 2, the reaction is very fast because the enzyme concentration in the SMBR is almost five times higher than that in the previous case and the sucrose concentration at the entrance of reaction zone is unchanged. The

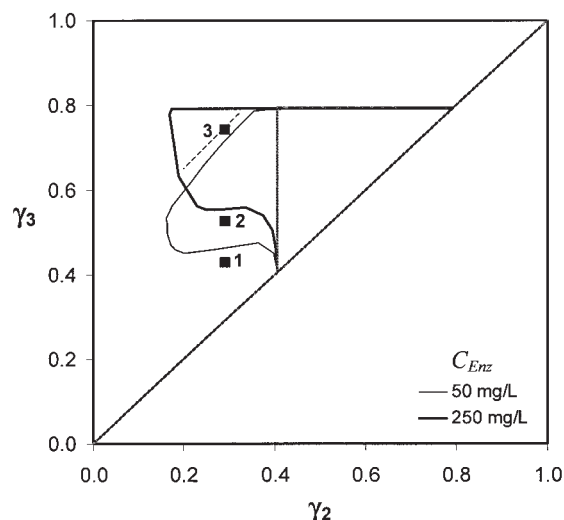
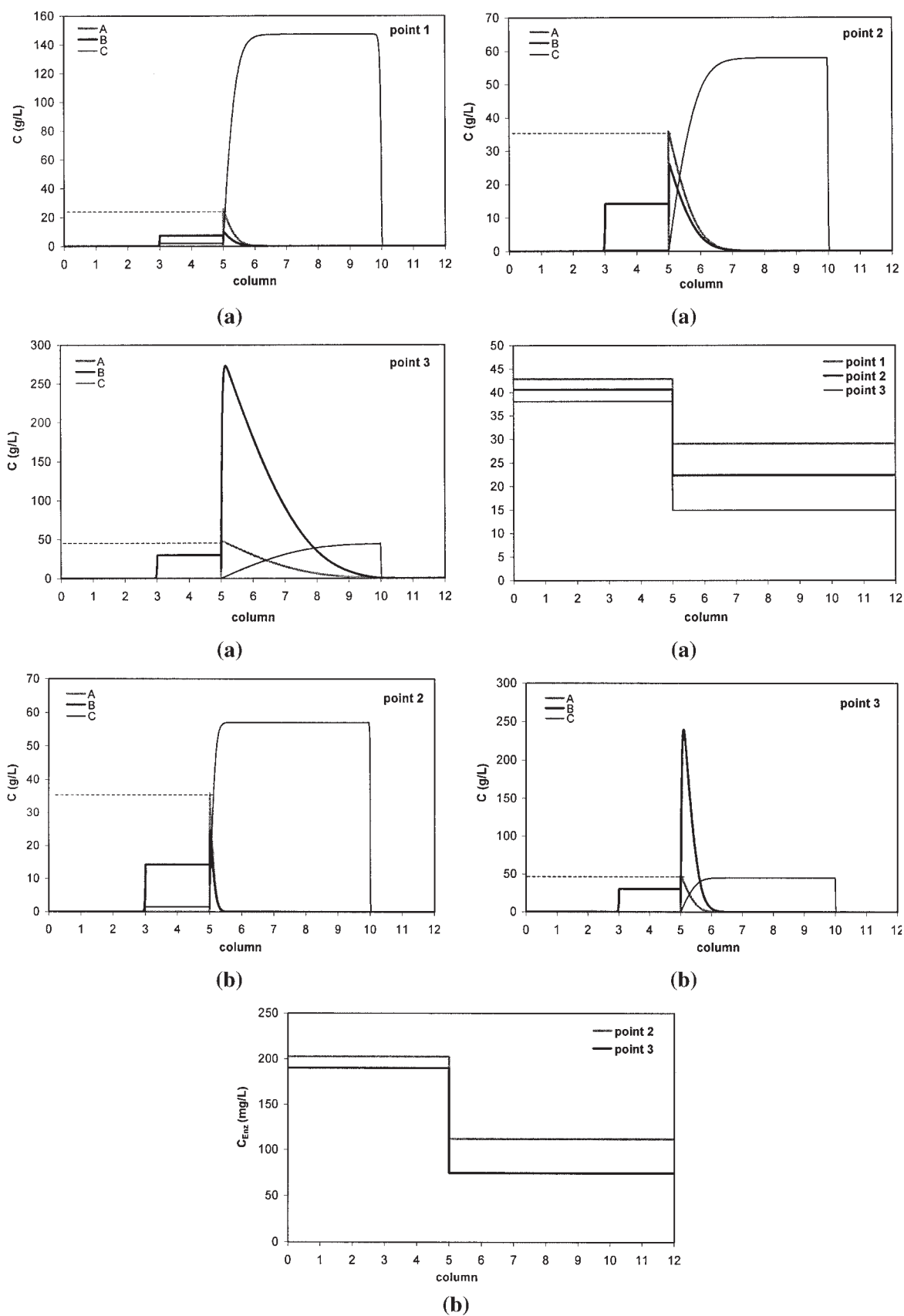


Figure 7. Reactive-separation regions for  $C_{ENZ} = 50$  mg/L and  $C_{ENZ} = 250$  mg/L with some selected points.

sucrose reacts completely very near to the feed port and the flow rate in zone 3 is not high enough to prevent the passage of glucose in zone 2. Also, the reaction is faster when  $C_{ENZ} = 250$  mg/L is used together with the operating conditions in point 3. The sucrose does not reach the raffinate port and pure raffinate is obtained. Point 2 is outside and point 3 is inside the reactive-separation region for  $C_{ENZ} = 250$  mg/L.

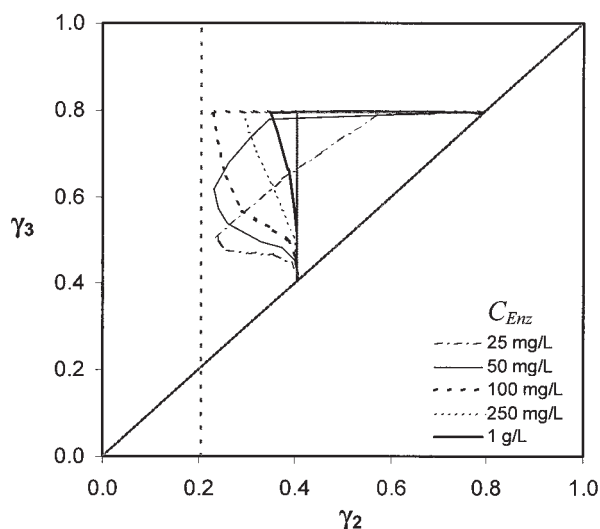
Let us see now why the reactive-separation region for  $C_{ENZ} = 50$  mg/L is cut along a line that corresponds to almost constant feed flow rate. It was shown before that point 3 does not belong to the reactive-separation region because of the contamination of the raffinate with sucrose and fructose, as a result of incomplete sucrose conversion (that is, insufficiently fast reaction) in zone 3. The same reason is valid for all points along the dashed line presented in Figure 7. The reason for the cutoff of the reactive-separation regions along the line with almost constant feed flow rate is the enzyme concentration in the eluent, which is not sufficiently high to promote complete sucrose conversion before the raffinate port. The points along the dashed line (Figure 7) would belong to the reactive-separation region when the enzyme concentration is increased from 50 to 100 mg/L, as demonstrated in Figure 5.

It would be interesting to analyze the obtained reactive-separation regions in terms of the maximum feed processed, product concentrations and the quantity of the enzyme used. The quantity of feed processed (or sucrose inverted) is directly connected to the difference of the net flow rates in zone 3 and 2 ( $\gamma_3 - \gamma_2$ ). So, we can say that the maximum feed will be processed for enzyme concentration around 250 mg/L (see



**Figure 8. Equivalent TMBR steady-state concentration profiles for points 1-3 from Figure 7.**

(a) Points 1-3 and  $C_{ENZ} = 50$  mg/L; (b) points 2 and 3 and  $C_{ENZ} = 250$  mg/L.



**Figure 9. Reactive-separation regions for the case  $K_A < K_C < K_B$  (in the absence of mass-transfer resistance) for different enzyme concentrations.**  
 $PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

Figure 5). The operating points that correspond to highest feed throughput belong to the region of low values  $\gamma_2$  (near  $\gamma_2 = 0.150$ ) and high values of  $\gamma_3$  ( $\gamma_3 = 0.795$ ). In that region, the products are quite diluted because the flow rates of the extract [which is directly connected to  $(\gamma_1 - \gamma_2)$ ] and raffinate [which is directly connected to  $(\gamma_3 - \gamma_4)$ ] are high. This would imply high cost for recovering the pure glucose and fructose from the extract and raffinate.

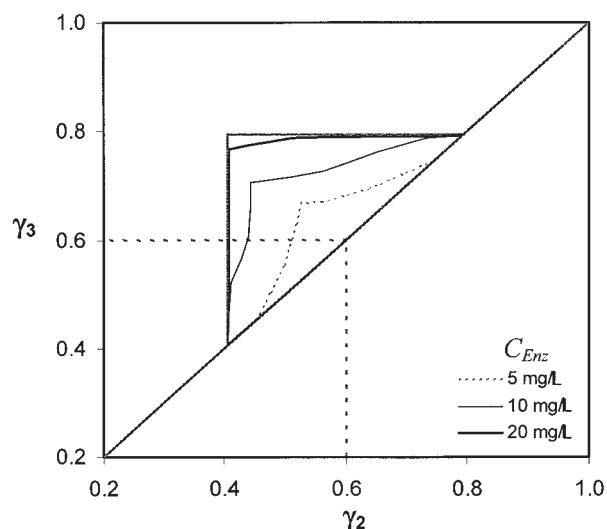
**Influence of the Reactant Henry Constant on the Reactive-Separation Regions.** In the SMBR unit for sucrose inversion, it is assumed that sucrose does not adsorb in the adsorbent particle. The Henry constant of sucrose is equal to zero; therefore, the minimum  $\gamma_2$  would be for  $\varepsilon = 0.4$  and  $\varepsilon_p = 0.1$ ,  $\gamma_{2\min} = \nu(K_A + \varepsilon_p) = 0.15$ . The reactive-separation region for all analyzed enzyme concentration also approaches a value of 0.15.

To better understand the connection between  $\gamma_2$  (or  $\gamma_3$ ) and the Henry constant of the reactant three hypothetical cases for the reactant adsorption parameter were considered:  $K_A = 0.037$ ,  $K_A = 0.3$ , and  $K_A = 0.57$ , or in terms of  $\gamma$  values,  $\gamma_A = 0.206$ ,  $\gamma_A = 0.600$ , and  $\gamma_A = 1.005$ . In all these cases it was assumed that the reactant enters in the adsorbent particle pores. The Henry constants of the products B and C were those used before:  $K_B = 0.43$  and  $K_C = 0.17$ , or in terms of  $\gamma$  values,  $\gamma_B = 0.795$  and  $\gamma_C = 0.405$ , respectively.

The reactive-separation regions for three cases for the reactant adsorption affinity relative to the products adsorption affinity were determined:

- (1) Reactant as less adsorbed component ( $K_A < K_C < K_B$ )
- (2) Reactant as middle adsorbed component ( $K_C < K_A < K_B$ )
- (3) Reactant as most adsorbed component ( $K_C < K_B < K_A$ )

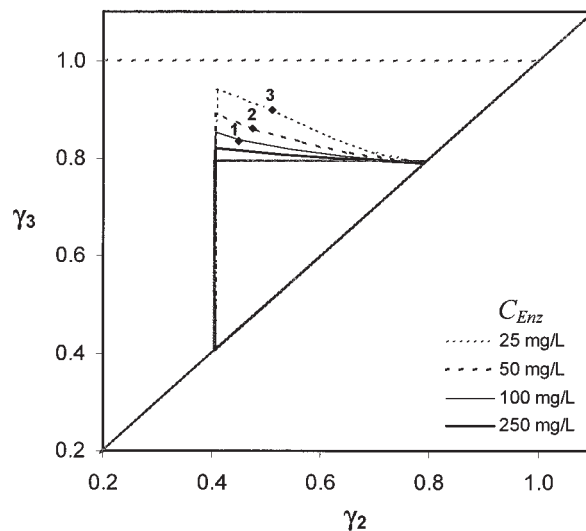
In Figure 9 the reactive-separation regions for the case  $K_A < K_C < K_B$  in the absence of mass-transfer resistance are presented. This case is very similar to the real one; the only difference is that now sucrose is adsorbed. Once again the reactive-separation region ap-



**Figure 10. Reactive-separation regions for the case  $K_C < K_A < K_B$  (in the absence of mass-transfer resistance) for different enzyme concentrations.**  
 $PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

proaches the line  $\gamma_2 = 0.206$ , which corresponds to the one calculated for the reactant A according to the equilibrium theory ( $\gamma_A = 0.206$ ).

The reactive-separation regions for the case of reactant as middle adsorbed component ( $K_C < K_A < K_B$ ) are presented in Figure 10. The reaction occurs in both zones 2 and 3 and the SMBR configuration used was 3–3–3–3. The reaction zone (zones 2 and 3) has one column less than in the case  $K_A < K_C < K_B$ . This configuration was selected because the value of  $K_C$  (0.205) is exactly the middle value between  $K_A$  (0.795) and  $K_B$  (0.405), and therefore, neither of zones 2 or 3 is privileged in



**Figure 11. Reactive-separation regions for the case  $K_B < K_C < K_A$  (in the absence of mass-transfer resistance) for different enzyme concentrations.**  
 $PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

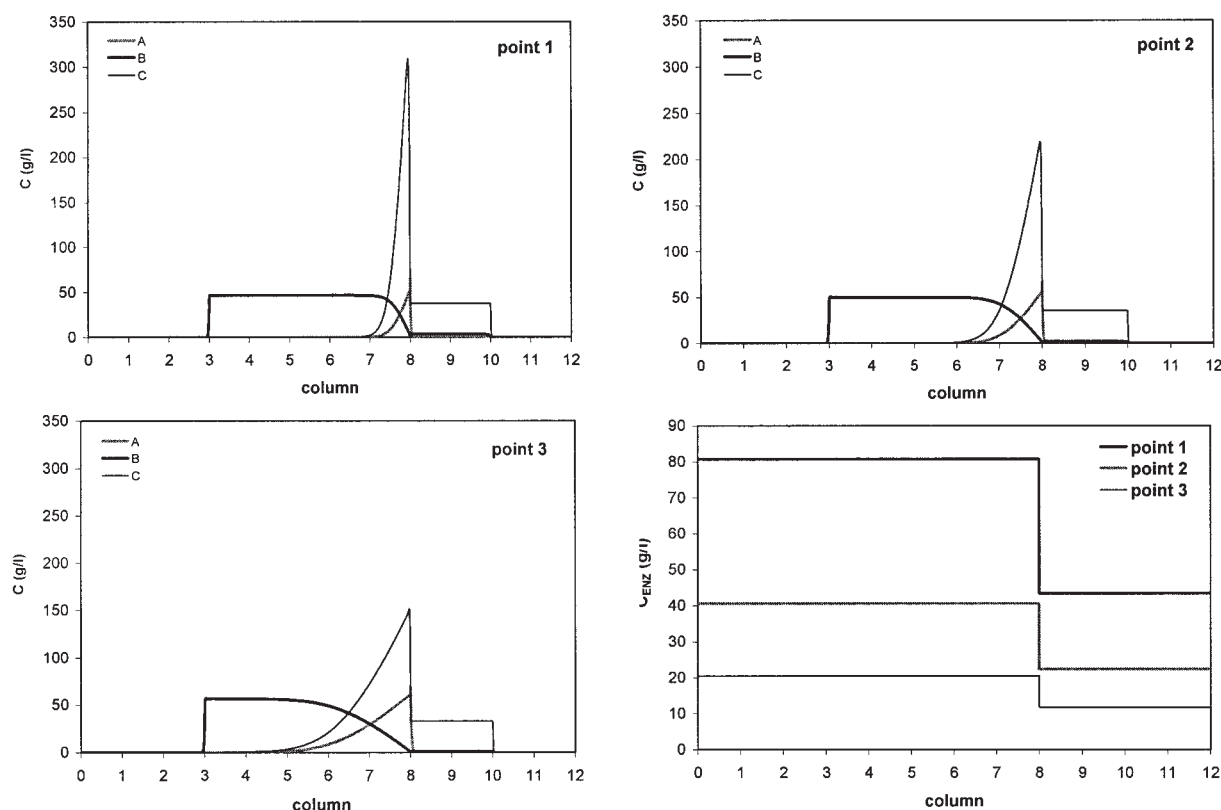


Figure 12. Equivalent TMBR steady-state concentration profiles for points 1–3.

terms of the reaction. The enzyme concentration was varied between 5 and 20 mg/L. For enzyme concentration above 20 mg/L the reactive–separation region matches with the separation region of products B and C. It can be seen that enzyme concentrations lower than those in the previous case are used.

For  $K_C < K_B < K_A$  the reactive–separation regions for different enzyme concentrations are presented in Figure 11. The 3–5–2–2 configuration was used because the reaction takes place just in zone 2. The enzyme concentration used in the calculation was between 25 and 250 mg/L. For all enzyme concentrations the border of the reactive–separation regions (with 99% extract end raffinate purities) with the region of pure raffinate only is  $\gamma_2 = 0.405$ . On the other hand, the border with the region of pure extract extends only to values of  $\gamma_3 > 0.795$ . When higher enzyme concentrations are used the reaction is fast and occurs at the end of zone 2, near the feed port; thus the system behaves closely to the nonreactive SMB for B/C separation.

The internal steady-state concentration profiles for points 1, 2, and 3 (see Figure 11) are presented in Figure 12. These points correspond to equal feed flow rates, but different enzyme concentrations: point 1,  $C_{Enz} = 100$  mg/L; point 2,  $C_{Enz} = 50$  mg/L; and point 3,  $C_{Enz} = 25$  mg/L. It can be seen that when the enzyme concentration decreases the reaction becomes slower, both B and C are produced more distant from the feed port, and thus a higher flow rate in zone 2 has to be used to prevent the contamination of the extract ( $>1\%$ ) with the product C. The flow rate in zone 3 would also be higher because the feed flow rate is constant for all three points. The opposite will

occur when the reaction is fast (point 1); lower values of flow rate in zones 2 and 3 have to be used to ensure the adsorption of component B and prevent the contamination of the raffinate.

Once again the extension of the reactive–separation region out of the nonreactive separation for the cases  $K_A < K_C < K_B$  and  $K_C < K_B < K_A$  is related with the reaction rate and small contamination of the products allowed. In both cases significantly higher feed throughput can be used. Also this throughput can be processed with the lower values of the enzyme concentration used, 100 mg/L in the case  $K_A < K_C < K_B$  and 25 mg/L in the case  $K_C < K_B < K_A$ . Additional analysis would be needed in terms of product concentration, that is, the costs for the recovery of products (B and C) from the extract and raffinate, respectively.

The SMBR for sucrose inversion is quite complex because the enzyme is not immobilized and its concentration in zones 1 and 2 is different from that in zones 3 and 4 because of the dilution with the feed stream. Therefore, even when a constant enzyme concentration in the eluent is used, any point in the reactive–separation region is characterized by different enzyme concentrations, that is, different reaction rates. Also, the reaction kinetic is described with the Michaelis–Menten equation, according to which the rate of the reaction changes both with enzyme and sucrose concentration in the reaction zone. All this introduces some complexity to the analysis and prediction of the reactive–separation region.

In the next section some simplifications will be introduced for an SMBR with  $A \rightarrow B + C$  type of reaction.

**Table 4. LDF Steady-State Equivalent TMBR Model Equations**

Mass balance in the inter-particle liquid phase in zone $j$	
$\frac{\gamma_j}{Pe_j} \frac{d^2 C_{i,j}}{dx^2} - \gamma_j \frac{dC_{i,j}}{dx} - \nu \alpha_{h_{i,j}} (K'_i C_{i,j} - \langle q \rangle_{i,j}) + \sigma_i Da_j C_{A,j} = 0$ $(i = A, B, C)$	
where	
$\sigma_i = \begin{cases} -1 & \text{if } i \text{ is } A \\ +0.526 & \text{if } i \text{ is } B \text{ or } C \end{cases}$	
Mass balance in the adsorbent particles in zone $j$	
$\frac{\partial \langle q \rangle_{i,j}}{\partial x} + \alpha_{h_{i,j}} [K'_i \langle C \rangle_{i,j} - \langle q \rangle_{i,j}] = 0$ $K' = K + \varepsilon_p$	
Boundary conditions for a given zone $j$ are	
$C_{i,j}^{\text{in}} = C_{i,j}(0) - \frac{1}{Pe_j} \frac{\partial C_{i,j}}{\partial x} \quad \frac{\partial C_{i,j}}{\partial x}(1) = 0 \quad (i = A, B, C)$ $\langle q \rangle_{i,j}(1) = \langle q \rangle_{i,j+1}(0) \quad (i = A, B, C)$	
Dimensionless groups	
$\gamma_j = \frac{U'_{F,j}}{U_S} \quad Pe_j = \frac{U'_{F,j} L_j}{D_{axj}} \quad \nu = \frac{(1 - \varepsilon)}{\varepsilon}$ $\alpha_{h_{i,j}} = \frac{k_{h,i} L_j}{U_S} \quad \sigma_i = \frac{\text{Mol. weight of } B \text{ (or } C)}{\text{Mol. weight of } A} \quad Da_j = \frac{L_j k_r}{U_S}$	
Node mass balances	
Eluent node	Feed node
$C_{i,1}^{\text{in}} = \frac{Q'_1}{Q'_1} C_{i,4}(1) + \frac{Q_E}{Q'_1} C_{i,E}$	$C_{i,3}^{\text{in}} = \frac{Q'_2}{Q'_3} C_{i,2}(1) + \frac{Q_F}{Q'_3} C_{i,F}$
Extract node	Raffinate node
$C_{i,2}^{\text{in}} = C_{i,1}(1)$	$C_{i,4}^{\text{in}} = C_{i,3}(1)$

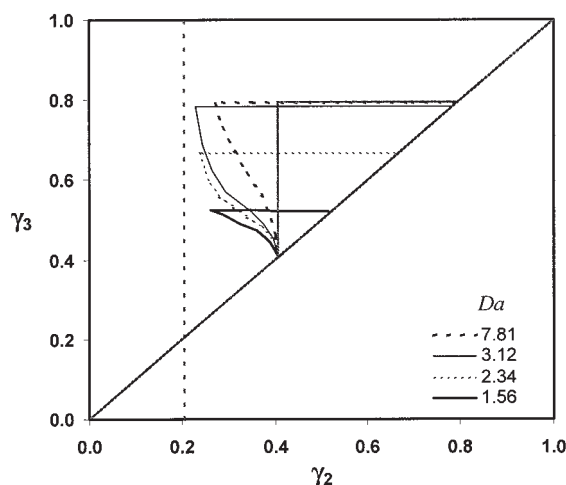
### SMBR for $A \rightarrow B + C$ : General Case

The SMBR where reactant A is irreversibly converted to products B and C ( $A \rightarrow B + C$ ) is considered. The reaction kinetics is of first-order linear reaction kinetics, catalyzed with enzyme immobilized on the adsorbent particles. All species are adsorbed and exhibit linear adsorption equilibrium isotherms.

For the purpose of modeling the SMBR was assumed as an equivalent TMBR (Figure 1b) operating in steady state. The mathematical model considers plug flow for the liquid and solid phases, the presence of internal mass transfer described by the linear driving force (LDF) model, and linear adsorption isotherm. This model is simpler than the bi-LDF steady-state TMBR model used previously. The LDF steady-state TMBR model equations are summarized in Table 4. Minceva et al.<sup>35</sup> recently proposed an analytical solution for this model.

**Table 5. LDF Steady-State Equivalent TMBR Model Input Parameters**

Model Parameters	Operating Conditions	Columns
$Pe = 500/\text{column}$ $k_r = 0.61 \text{ min}^{-1}$ $k_{hB} = 1.32 \text{ min}^{-1}$ $k_{hC} = 1.89 \text{ min}^{-1}$ $K_B = 0.43$ $K_C = 0.17$ $\varepsilon_p = 0.1$	$t^* = 3.4 \text{ min}$ $C_F = 80 \text{ g/L}$ $Q_{Rec} = Q_4 = 24 \text{ mL/min}$ $\gamma_1 = 0.953$ $\gamma_4 = 0.325$	$D_b = 2.6 \text{ cm}$ $L_b = 29 \text{ cm}$ Configuration: 3–3–3–3



**Figure 13. Reactive-separation regions for first-order reaction  $A \rightarrow B + C$ , in the case  $K_A < K_C < K_B$  (in the absence of mass-transfer resistance) for different  $Da$  numbers.**

$PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

The SMBR configuration, column geometry, and operating conditions, together with the adsorption equilibrium, mass transfer, and reaction parameters for the reactant A and products B and C are presented in Table 5.

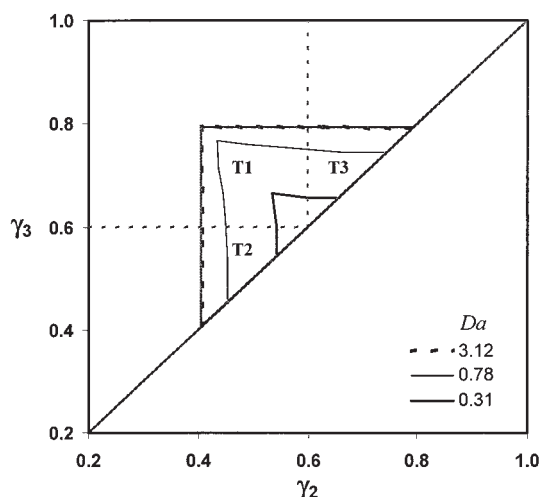
The reactive-separation regions were determined in the absence of mass-transfer limitation; the internal mass transfer and axial dispersion were eliminated by multiplying the mass-transfer parameter and  $Pe$  number by a factor of 100. The product purities were set to 99% and the conversion at a minimum of 99%.

Three different cases for the reactant adsorption affinity relative to those of products B and C were considered:

- (1) Reactant as less adsorbed component ( $K_A < K_C < K_B$ )
- (2) Reactant as middle adsorbed component ( $K_C < K_A < K_B$ )
- (3) Reactant as most adsorbed component ( $K_C < K_B < K_A$ )

The Henry constants of the products B and C were  $K_B = 0.43$  ( $\gamma_B = 0.795$ ) and  $K_C = 0.17$  ( $\gamma_C = 0.405$ ). The reactant A adsorption parameters were:  $K_A = 0.037$ ,  $K_A = 0.3$ , and  $K_A = 0.57$ , or  $\gamma_A = 0.206$ ,  $\gamma_A = 0.600$ , and  $\gamma_A = 1.005$ , respectively. For all three cases of reactant adsorption affinity the SMBR configuration 3–3–3–3 was selected and the  $\gamma_1$  and  $\gamma_4$  were set on 0.953 and 0.325, respectively.

The reactive-separation regions for the case of reactant as less adsorbed component,  $K_A < K_C < K_B$  ( $0.037 < 0.17 < 0.43$ ), for different Damköhler ( $Da$ ) numbers are presented in Figure 13. The reactant A is carried out with the liquid phase in the direction of the raffinate port and the reaction occurs strictly in zone 3 and the  $Da$  number in this zone is thus the



**Figure 14. Reactive-separation regions for first-order reaction  $A \rightarrow B + C$ , in the case  $K_C < K_A < K_B$  (in the absence of mass-transfer resistance) for different  $Da$  numbers.**

$PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

controlling parameter. The reactive-separation regions were determined for  $Da$  numbers 1.56, 2.34, 3.12, and 7.81 (see Figure 13). For all values of the  $Da$  number the reactive-separation regions approaches the line  $\gamma_2 = 0.206$ , which corresponds to  $\gamma_A = 0.206$ , calculated according to the equilibrium theory. The border of the region of pure extract only is defined by a constant value of  $\gamma_3$ , which increases with the increase of  $Da$  number and approaches  $\gamma_3 = \gamma_B = 0.795$  when the reaction is fast enough. For low values of  $Da$  the reaction is slow, and a lower flow rate in zone 3 must be used to increase the residence time of the reactant  $A$  and ensure conversion higher than 99% and prevent the contamination of the raffinate with the product  $B$ . With increasing  $Da$  number the reaction becomes faster and a higher flow rate in zone 3 can be used. Once again, in the case of sufficiently high  $Da$  numbers, the reactant is completely converted near the feed port and the reactive separation region matches the equivalent nonreactive SMB separation region.

In Figure 14 the reactive-separation regions for the case  $K_C < K_A < K_B$  ( $0.17 < 0.3 < 0.43$ ) for different  $Da$  numbers 0.31, 0.78, and 3.12 are presented. The reaction takes place in zone 2 (region T2), zone 3 (region T3), and both zones 2 and 3 (region T1). For  $Da = 3.12$  the reactive-separation region matches the separation region of products  $B$  and  $C$ . The case of middle adsorbed reactant is analyzed in detail by Fricke and Schmidt-Traub<sup>33</sup> and Lode et al.<sup>34</sup> They developed explicit design criteria for the reactant conversion as a function of the flow rate ratios as well as of the  $Da$  numbers within zones 2 and 3 for a constant degree of reactant conversion.

The influence of the  $Da$  number (that is, reaction rate constant) on the reactive-separation regions shape and position for the case  $K_C < K_B < K_A$  ( $0.17 < 0.43 < 0.57$ ) is presented in Figure 15. The reactant, as most adsorbed component, is carried with the solid phase in the direction of the extract port and thus does not enter zone 3. In the absence of mass-transfer resistance the reaction takes place just in zone 2; the  $Da$  number in this zone is thus the controlling parameter. The

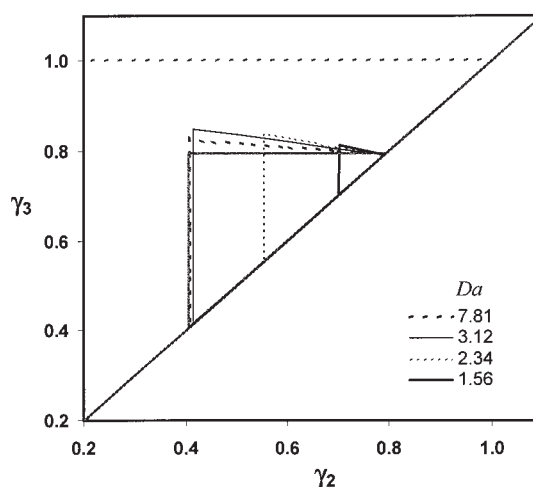
reactive-separation regions were determined for  $Da$  numbers 1.56, 2.34, 3.12, and 7.81 (see Figure 15). The border of the reactive-separation regions with the region of pure raffinate only is defined by a constant value of  $\gamma_2$ , which decreases with the increment of  $Da$  number and approaches  $\gamma_2 = \gamma_C = 0.405$  when the reaction becomes fast enough. When the reaction is slow (low values of  $Da$ ) a higher flow rate in zone 2 must be used to prevent the contamination of the extract with the product  $C$ . On other hand with increasing  $Da$  number the reaction becomes faster and a lower flow rate in zone 2 can be used.

The border of the reactive-separation regions with the region of pure extract only goes beyond the  $\gamma_3$  value calculated from the equilibrium theory for nonreactive SMB, that is,  $\gamma_3 = \gamma_B = 0.795$ . Because the product purities are set at 99%, a higher flow rate in zone 3 can be used.

## Conclusions

The design of a simulated moving-bed reactor (SMBR) for an irreversible reaction,  $A \rightarrow B + C$ , was studied. The steady-state equivalent true moving-bed reactor (TMBR) analogy was implemented in the algorithm used for determination of the reactive-separation regions. Two reactive systems were considered: (1) sucrose inversion, with enzyme introduced in the unit through the eluent stream and Michaelis-Menten reaction kinetics, and (2)  $A \rightarrow B + C$  reaction, with immobilized enzyme and linear reaction kinetic law.

The bi-LDF equivalent TMBR model was used to study the product purities and the effect of enzyme concentration on the reactive-separation regions of SMBR for sucrose inversion in the presence of mass-transfer resistance. Because of the relaxation of the product purity requirements the reactive-separation region expands out of the nonreactive SMB separation region, in the direction of lower values of net-flow rate in zones 2 and 3. The same feed throughput could be processed for different enzyme concentrations and constant product purities constraint (95%). This information is very important for the minimization



**Figure 15. Reactive-separation regions for first-order reaction  $A \rightarrow B + C$ , in the case  $K_C < K_B < K_A$  (in the absence of mass-transfer resistance) for different  $Da$  numbers.**

$PUX = 99\%$ ,  $PUR = 99\%$ , and  $X_{\min} = 99\%$ .

of the enzyme inventory. The reactive–separation regions were also determined in equilibrium conditions (absence of mass-transfer resistance). Under this condition the regions spread to values of  $\gamma_2$  near  $\gamma_{2\min} = \nu(K_A + \varepsilon_p) = 0.150$  with the increase of the enzyme concentration in the eluent. The main reasons for this behavior are the rate of sucrose conversion in the reaction zone (zone 3) and the small allowed impurity (1%) in the extract and raffinate streams. The reactant adsorption affinity relative to that of the products influences the shape and the position of the reactive–separation region.

In the case of SMBR for general reaction  $A \rightarrow B + C$ , with immobilized enzyme and linear reaction kinetic law, special attention was given to the influence of the reactant (A) adsorption affinity on the reactive–separation regions. In the absence of mass-transfer resistance and for given conversion and product purity, the size of the reactive–separation regions is governed by the rate of the reaction. In the cases of reactant (A) as the less adsorbed and the most adsorbed species the regions expand out of the separation regions in the direction of lower  $\gamma_2$  and higher  $\gamma_3$  values, respectively. Once again the expansion of the reactive–separation regions out of the equivalent nonreactive SMB for products B/C separation results from the rate of the reaction and small impurity allowed in the product streams.

## Acknowledgments

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## Notation

$Bi_m$	= mass Biot number
$C$	= bulk fluid phase concentration, kg/m <sup>3</sup>
$\langle C_p \rangle$	= pore fluid phase concentration averaged over the particle volume, kg/m <sup>3</sup>
$Da$	= Damköhler number
$D_{ax}$	= axial dispersion coefficient, m <sup>2</sup> /s
$D_b$	= column diameter, m
$d_p$	= diameter of adsorbent particle, m
$k$	= mass-transfer constant, s <sup>-1</sup>
$K$	= adsorption constant as defined in the bi-LDF approximation
$K'$	= adsorption constant as defined in the LDF approximation
$k_f$	= film mass-transfer coefficient, m/s
$k_r$	= rate constant of the inversion reaction kinetics, kg <sub>A</sub> /kg <sub>Enz</sub> s <sup>-1</sup>
$K_{enz}$	= adsorption constant of enzyme invertase onto adsorbent resin
$K_{mm}$	= Michaelis–Menten constant, kg/m <sup>3</sup>
$L_b$	= column length, m
$L_j$	= zone length, m
$Pe$	= Peclet number
$PRA$	= adsorbent productivity, kg m <sup>-3</sup> h <sup>-1</sup>
$PRE$	= enzyme productivity, kg <sub>A</sub> /g <sub>Enz</sub>
$PUR$	= raffinate purity, %
$PUX$	= extract purity, %
$\langle q \rangle$	= adsorbed phase concentration averaged over a homogeneous solid, kg/m <sup>3</sup>
$\langle\langle q \rangle\rangle$	= adsorbed phase concentration averaged over a bidisperse particle, kg/m <sup>3</sup>
$Q$	= volumetric flow rate in a SMBR, m <sup>3</sup> /s
$Q'$	= volumetric flow rate in a TMBR, m <sup>3</sup> /s
$R_j$	= reaction rate law, kg m <sup>-3</sup> s <sup>-1</sup>
$R_p$	= particle radius, m
$t$	= time, s
$t^*$	= switching time or rotation period, s
$T$	= temperature, K
$U_F$	= interstitial fluid velocity in a SMBR, m/s
$U'_F$	= interstitial fluid velocity in a TMBR, m/s
$U_S$	= interstitial solid velocity in a TMBR, m/s
$x$	= dimensionless space co-ordinate in TMBR-based model
$X$	= reaction conversion, %

## Greek letters

$\alpha$	= number of mass-transfer or reaction rate units
$\gamma$	= ratio between fluid and solid interstitial velocities in a TMBR
$\varepsilon$	= bed porosity
$\varepsilon_p$	= adsorbent particle porosity
$\nu$	= ratio between solid and fluid volumes in a column
$\sigma$	= stoichiometric reaction parameter

## Indices

1, . . . , 4	= index to indicate TMB/SMB zones
A	= sucrose
B	= fructose
C	= glucose
Enz	= enzyme
E	= eluent
F	= feed
h	= referring to a homogeneous solid
i	= chemical species
in	= at the inlet of a column (superscript)
j	= TMB zone
p	= referring to the pores of a bidisperse adsorbent
r	= reaction
R	= raffinate
X	= extract
$\mu$	= referring to the microparticles of a bidisperse adsorbent

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